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HEAT TRANSFER IN A CHANNEL AT SUPERCRITICAL PRESSURE

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Abstract—The paper concerns a theoretical analysis of the convective heat transfer at supercritical pressures in a channel. The analysis is based on the flow division into two zones with averaged properties the interface between them being the surface of the pseudocritical temperature. Hence, the conservation equations of momentum and energy may be solved separately resulting in analytic formulas for the velocity and temperature profiles and the Nusselt number. The theoretical results are plotted against own experiments with CO_2 revealing a fairly good agreement.

NOMENCLATURE

c_p,	specific heat at constant pressure;
d,	inside tube diameter;
g,	gravitational acceleration;
i,	specific enthalpy;
k,	thermal conductivity coefficient;
m,	mass flow rate;
p,	pressure;
q,	heat flux;
r,	radial coordinate (from the axis);
u,	axial velocity;
y,	radial coordinate (from the wall);
z,	axial coordinate;
T,	temperature;
$Y = 2y/d$,	dimensionless radial coordinate;
δ,	dimensionless thickness of the laminar sublayer;
ε,	turbulent diffusivity;
μ,	dynamic viscosity coefficient;
ν,	kinematic viscosity coefficient;
ρ,	density;
τ,	shear stress.

Subscripts

e,	experiment;
h,	heavy zone;
in,	inlet;
l,	laminar flow; light zone;
pc,	pseudo-critical state;
q,	heat transfer;
t,	turbulent flow; theory;
w,	wall;
τ,	momentum transfer.

Superscripts

$^+$,	dimensionless;
$^-$,	mean value.

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INTRODUCTION

HEAT TRANSFER at supercritical pressures has become a subject of growing interest in the last 20 years along with the rapid development of energy, rocket and cryogenic technology. Many experimental and theoretical investigations made in this period have been concentrated on the specific features of supercritical pressure heat transfer, i.e. crises, local augmentation of heat transfer and oscillation of pressure and flow rate under certain conditions. Essentially the problem of convective heat transfer at supercritical pressure concerns the determination of velocity and temperature profiles in the fluid of variable, strongly temperature dependent properties characteristic for the fluid in the near critical state (Fig. 1). This fact must inevitably be taken into consideration in studies on this heat transfer problem. Strongest variations of all the thermophysical properties occur near the pseudocritical temperature T_{pc} which for a given fluid is a function of pressure alone (similarly as the saturation temperature at subcritical pressures). Specific heat and thermal conductivity reach their maxima at this temperature and density and viscosity reveal maximal variations.

FORMULATION OF THE PROBLEM

The analysis is performed for the supercritical pressure upward flow in a vertical, uniformly heated, circular tube. Turbulent, axially symmetrical, fully developed and steady flow with negligible axial heat conduction and viscous dissipation is assumed. The effect of pressure drop on the fluid properties is also neglected. Under above assumptions the equations of momentum and energy have a form, respectively:

$$\rho u \frac{\partial u}{\partial z} = -\frac{\partial p}{\partial z} - \frac{1}{r}\frac{\partial}{\partial r}(r\tau) - \rho g, \quad (1)$$

$$\rho u \frac{\partial i}{\partial z} = \frac{1}{r}\frac{\partial}{\partial r}(rq), \quad (2)$$

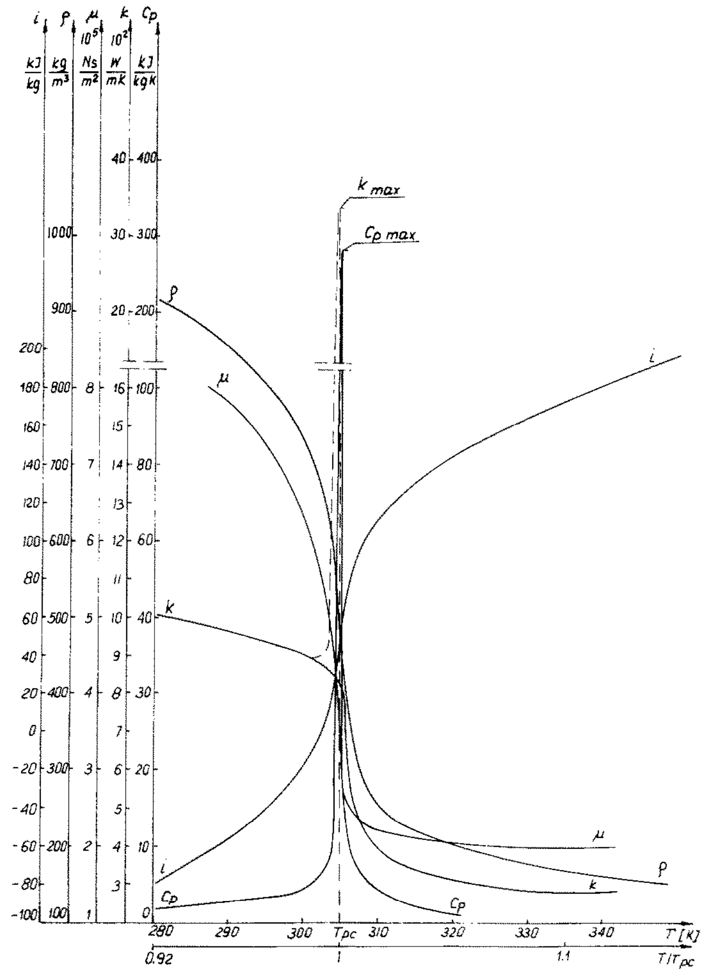

FIG. 1. Thermophysical properties of CO_2 in the near-critical region ($p = 7.5$ MPa, $p/p_c = 1.02$). Data from [1].

and the appropriate boundary conditions are:

$$r = 0 \quad \tau = 0 \quad \partial u/\partial r = 0, \tag{1a}$$

$$r = R \quad \tau = \tau_w \quad u = 0, \tag{1b}$$

$$r = 0 \quad q = 0 \quad \partial T/\partial r = 0, \tag{2a}$$

$$r = R \quad q = q_w \quad T = T_w. \tag{2b}$$

Dimensionless velocity and temperature profiles of a turbulent flow are coupled with the shear stress, heat flux and turbulent diffusivity distributions along the channel radius in a manner given by

$$u^+ = \int_0^{y^+} \frac{\tau/\tau_w}{1+\varepsilon_\tau^+} \, dy^+, \tag{3}$$

$$T^+ = \int_0^{y^+} \frac{q/q_w}{1+\varepsilon_\tau^+ \, Pr/Pr_t} \, dy^+, \tag{4}$$

where

$$u^+ = u/\sqrt{(\tau_w/\rho_0)}, \tag{5}$$

$$T^+ = (T_w - T)\, k_0 \sqrt{(\tau_w/\rho_0)}/q_w \, v_0, \tag{6}$$

$$y^+ = y\sqrt{(\tau_w/\rho_0)}/v_0, \tag{7}$$

$$\varepsilon_\tau^+ = \varepsilon_\tau/v_0, \quad Pr_t = \varepsilon_q^+/\varepsilon_\tau^+. \tag{8}$$

Determination of the velocity and temperature profiles in a channel and, in turn, the shear stress and temperature at the wall requires the solution of the set of non-linear differential equations (1–2) with the turbulent transport parameters ε_τ^+ and Pr_t defined in a certain way.

To date, this problem has not been solved analytically, and in important and quite extensive literature one can find a number of numerical solutions with different levels of simplification. Following the relatively comprehensive and critical review of references contained in [15] one can point out some characteristic features of previous approaches to the problem under consideration.

In the early papers [2–4] velocity and temperature profiles were calculated without referring to the conservation equations. The calculations were based on the assumed heat flux density and shear stress distributions across the channel, e.g. $\tau/\tau_w = 1$, $\tau/\tau_w = r/R$. The turbulent transport mechanism was described by the formulae originally deduced for constant property fluids and, in different manners, adapted to variable properties.

Very simple but descriptive analyses of the shear stress distribution in a variable property flow are presented in [5, 6], where a possibility of the shear stress reduction to zero near the wall due to buoyancy forces or an injection effect is shown, respectively.

A long series of investigations involving a fundamental approach based on the conservation equations started in the early 1960's [7, 8] and is continued along with the progress in numerical methods and electronic computers. These investigations reveal natural trends towards the solution of the conservation equations in their still more complex form and towards better understanding the turbulent transport in variable property fluids. Some recent papers are here a good example.

In [9] the authors compared nine different semi-empirical models (or their versions) of turbulence. The models were used in the numerical solution of the set of the conservation equations written in the form allowing for mixed convection, compressibility and viscous dissipation. The predictions were plotted against experimental data in this way exhibiting the applicability of different models. This comparison may be of help in selecting appropriate semi-empirical models of turbulence when a variable property case is considered.

Another approach to the description of turbulence and its relation to variable transport properties, is presented in [10]. The turbulent diffusivity of momentum and the turbulent Prandtl number in variable property fluids are expressed in terms of the differential equations of turbulent kinetic energy and enthalpy pulsations, respectively. However, the equations contain some experimental parameters originally determined for constant property fluids. Using the above model of turbulence the authors solved 2-D conservation equations with the gravity term in the momentum equation and obtained a good agreement with the experimental data for water and air. The same procedure was next repeated in [11], where the calculations for supercritical water and carbon dioxide were performed. The method proved to be fairly accurate in that case also.

A simple method of adapting a constant-property semi-empirical momentum diffusivity to the variable property case was presented and applied in [12, 13]. The adaptation was based on the shear stress correction factor resulting from the momentum equation. A set of the 2-D conservation equations was solved numerically in the paper. However, the gravity term was omitted thus limiting the validity of the method to forced convection only.

A theoretical analysis of combined forced and free convection heat transfer for turbulent flow of supercritical fluids was presented in [14] which was based on the principle of surface renewal. This relatively new approach follows the instantaneous energy and momentum equations associated with the unsteady transport to individual elements of fluid in residence near the wall. Also this approach leads to predictions for the mean transport properties which are consistent with experimental observations, claim the authors.

A common feature of recent theoretical studies on variable property heat transfer is the extensive use of numerical methods. Obviously, they allow the solving of complex sets of partial differential equations but usually employ large computers and much programming work. This paper presents a new simplified model of supercritical pressure heat transfer and the corresponding analytic method which results in comprehensive flow characteristics with the effect of buoyancy forces. The details of the method and experiments are given in [15].

THE MODEL

Inspection of Fig. 1 suggests that within the temperatures $T_{in} < T_{pc} < T_w$ there exist in the channel cross-section practically two zones of significantly different properties. Low density, viscosity and thermal conductivity fluid layer of the temperature within $\langle T_{pc}, T_w\rangle$ attaches the wall. This area is further referred to as the "light" zone. In contrast to this layer the flow core is occupied by the fluid of significantly higher density, viscosity and thermal conductivity thus being referred to as the "heavy" zone. Its temperature varies between T_{in} and T_{pc}. The interface between the both zones is an isothermal rotational surface of the pseudo-critical temperature T_{pc}. The idea of the flow divided into two zones is shown in Fig. 2.

So far as the heavy zone is concerned it is assumed that the total mass flow rate in this zone remains the same as if the constant property turbulent velocity profile was valid there. On the other hand a substantial difference in enthalpy between the two zones suggests that the heat transferred from the wall is entirely accumulated within the light zone. The second assumption allows one to determine the mass flow rate within the light zone similarly to flows with evaporation:

$$m_l = \frac{\pi d q_w (z - z_{in})}{i_l - i_h}. \tag{9}$$

From the mass balance one obtains $m_h = m - m_l$ and accordingly the first assumption on the velocity profile in the heavy zone the cross-section areas occupied by the both zones and thus the geometric coordinate of the interface $r_{pc}(y_{pc})$ may be determined.

The concept of two zones of constant average properties in each zone allows to solve equations (1) and (2) independently and substantially simplifies the problem under consideration. Equation (1) is firstly integrated over the channel area A, the result put back into this equation and the pressure gradient $\mathrm{d}p/\mathrm{d}z$ thus eliminated. As a result the differential equation of the shear stress distribution in the channel is obtained

$$\frac{1}{r}\frac{\mathrm{d}}{\mathrm{d}r}(r\tau) = \frac{2\tau_w}{R} + g(\bar{\rho} - \rho), \tag{10}$$

where

$$\bar{\rho} = \frac{1}{A}\int_0^A \rho \mathrm{d}A.$$

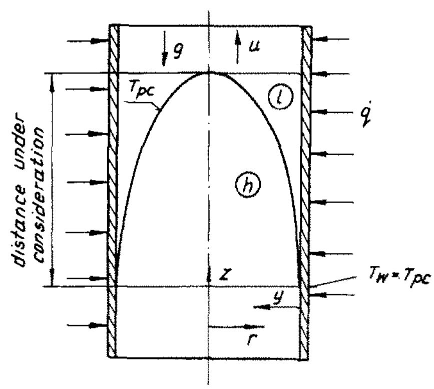

FIG. 2. The two-zone model of supercritical pressure flow.

Solving equation (10) with the boundary conditions (equations 1a, b) and non-dimensionalizing the result one gets the shear stress distribution in both zones:

$$\tau_l^+ = \frac{\tau_l}{\tau_w} = D(Re^* - y^+) + E\frac{1}{Re^* - y^+}, \quad (11)$$

$$\tau_h^+ = \frac{\tau_h}{\tau_w} = F(R^+ - y^+), \quad (12)$$

where

$$Re^* = \frac{R\sqrt{(\tau_w/\rho_l)}}{\nu_l}, \quad (13)$$

$$R^+ = Re^*\left[Y_{pc} + (1 - Y_{pc})\sqrt{\left(\frac{\rho_l}{\rho_h}\right)\frac{\nu_l}{\nu_h}}\right], \quad (14)$$

$$Re = \frac{R\bar{u}}{\nu_l}, \quad (15)$$

$$D = \left[\frac{2\tau_w}{R} + g(\bar{\rho} - \rho_l)\right]\frac{R}{\rho_l \bar{u}^2}\cdot\frac{Re^2}{2Re^{*3}}, \quad (16)$$

$$E = (1 - D\,Re^*)\,Re^*, \quad (17)$$

$$F = \left[D(Re^* - y_{pc}^+) + \frac{E}{Re^* - y_{pc}^+}\right]\frac{1}{R^+ - y_{pc}^+}. \quad (18)$$

The turbulent diffusivity coefficient was estimated after Prandtl's model as a first approximation of a much more complicated phenomenon:

$$\text{for} \quad y^+ \leqslant \delta, \quad \varepsilon_\tau^+ = 0, \quad (19)$$

and

$$\text{for} \quad y^+ > \delta, \quad \varepsilon_\tau^+ = 0.4y^+. \quad (20)$$

The influence of variable properties on the turbulent diffusivity was taken into account after Goldmann [2] by the relevant definition of the non-dimensional coordinate y_{pc}^+:

$$y^+ = \int_0^y \frac{\sqrt{(\tau_w/\rho)}}{\nu}\,dy. \quad (21)$$

With the two zone model this coordinate is easily calculated.

By introducing equations (11, 12) and (19, 20) into equation (3) and integrating three analytic non-dimensional formulae for the complete velocity profile are yielded: u_{ll}^+ for the laminar sublayer, u_{lt}^+ for the turbulent core of the light zone and u_{ht}^+ for the turbulent heavy zone. The only remaining unknown shear stress parameter Re^* is numerically determined from algebraic equation (22) obtained by equalizing the known mass flow rate in non-dimensional form with the integral of the non-dimensional velocity profile already predicted:

$$\frac{m_l\sqrt{(\tau_w/\rho_l)}}{2\pi\rho_l\nu_l^2} + \frac{m_h\sqrt{(\tau_w/\rho_h)}}{2\pi\rho_h\nu_h^2} = \int_0^{y_{pc}^+} u_l^+\,(Re^* - y^+)\,dy^+ + \int_{y_{pc}^+}^{R^+} u_h^+\,(R^+ - y^+)\,dy^+. \quad (22)$$

The heat flux distribution q/q_w

$$\frac{q}{q_w} = \frac{Re^*}{Re^* - y^+}\left[1 - \frac{2}{2Re^*\,y_{pc}^+ - y_{pc}^{+2}} \times \frac{\rho_l\sqrt{(\tau_w/\rho_l)}}{\overline{(\rho u)}_l}\int_0^{y^+} u^+(Re^* - y^+)\,dy^+\right], \quad (23)$$

and subsequently, the temperature profile within the light zone is determined from equation (2) with the boundary conditions equations (2a, b) by putting the velocity profile and integration. $Pr_t = 1$ was assumed in equation (4).

The Nusselt number may be conveniently defined with reference to the constant pseudo-critical temperature T_{pc}:

$$Nu = \frac{q_w}{T_w - T_{pc}}\frac{d}{k_l} = \frac{2Re^*}{T_{pc}^+}. \quad (24)$$

The wall temperature is then expressed by

$$T_w = T_{pc} + T_{pc}^+\frac{q_w\nu_l}{k_l\sqrt{(\tau_w/\rho_l)}}. \quad (25)$$

EXPERIMENTAL VERIFICATION OF THE MODEL

For the sake of verification of the theoretical model the experiments were performed and, afterwards, the measured longitudinal wall temperature distributions were compared to the predicted ones [after equation (25)]. The layout of the test rig described in detail in [15] is shown in Fig. 3. The rig is basically a natural circulation loop made of the stainless steel tube of constant diameter within the whole loop. The tube is 10.6 mm in inside diameter with the wall 2.2 mm in thickness. The test section made of the same tube is 1 m in heated length between the bus-bars and is directly DC heated. Uniformly distributed along the section and welded outside the wall are 11 Fe–Ko coated electrically insulated thermocouples. The test section is carefully insulated and the heat losses of the evacuated loop had been measured before the main experiments started. The experiments were performed with CO_2 within the following range of parameters: p = 7.5–8.0 MPa; q = 27–110 kW/m^2; $u\rho$ = 220

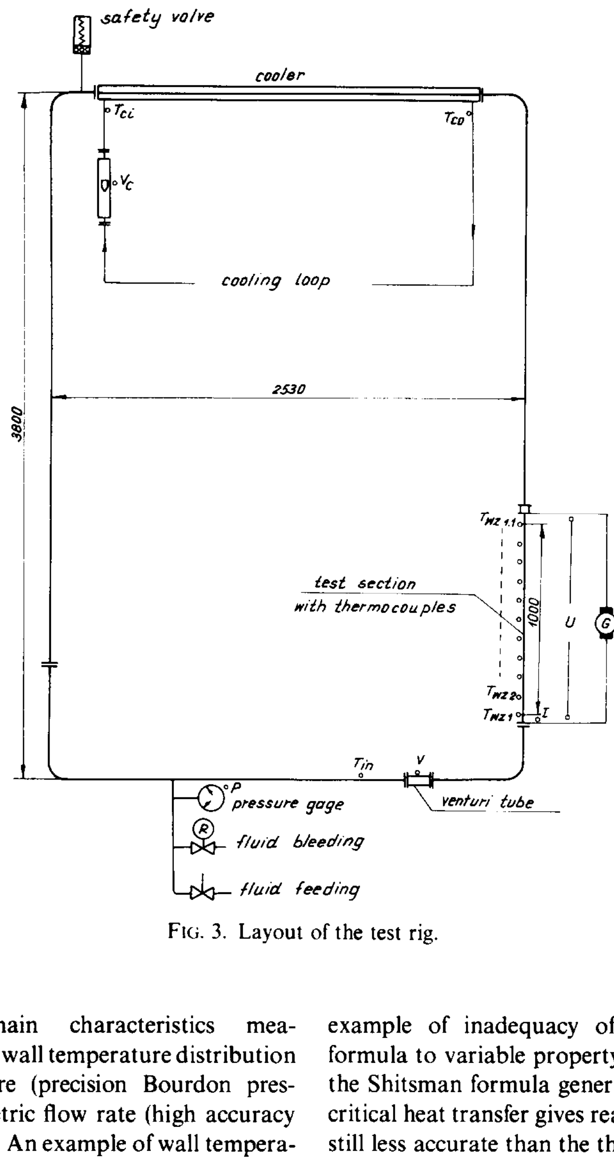

FIG. 3. Layout of the test rig.

-480 kg/m^{2}s. The main characteristics measured were: longitudinal wall temperature distribution (thermocouples), pressure (precision Bourdon pressure gauge) and volumetric flow rate (high accuracy Venturi tube flowmeter). An example of wall temperature distribution measured at constant pressure and variable heat flux is shown in Fig. 4.

The experimental Nusselt numbers obtained at maximal and minimal parameters controlling the phenomenon were plotted against their predicted values in Fig. 5. Two thirds of the predicted points fell within the error limit of $\pm 20\%$, and 3/4 of the points fell within $\pm 30\%$. However, the error distribution along the tube length is not uniform, see Fig. 6. The points appearing higher than the errors correspond either to the heat transfer crisis not accounted for by this model (and probably attributed to relaminarization) or to the uncertainty of the thermal conductivity data in the vicinity of the pseudo-critical temperature. Newer data revealing a local maximum of thermal conductivity in the pseudo-critical (region bounded by the dotted line in Fig. 1) allow a reduction in the error to -20%. The comparison between the results predicted after the theoretical model and three experimental formulae speaks in favour of the model. The Dittus–Boelter formula is involved only as a clear example of inadequacy of the constant property formula to variable property fluids. On the contrary, the Shitsman formula generated especially for supercritical heat transfer gives reasonably good results but still less accurate than the theory. The same holds for the much newer Protopopov formula [16] based on many experimental data and allowing for both the mixed convection and variable property effects.

Thickness of the laminar sublayer appears to be very important and it is recommended to take $\delta = 5$ except for the crisis cross-section.

Predicted velocity and temperature profiles at a given heat flux are presented in Fig. 7. In consistency with the assumption taken before, the temperature profiles span over the light zone only. The velocity profiles appear as distinct local maxima due to the buoyant forces.

SUMMARY

The theoretical model described in the paper allows one to determine the complete characteristics of the supercritical pressure flow with heat transfer under significant influence of the buoyancy forces. The model is based on an order of simplifications thus giving the solution an approximate character. All the characteristics are given in an analytic form except for the shear

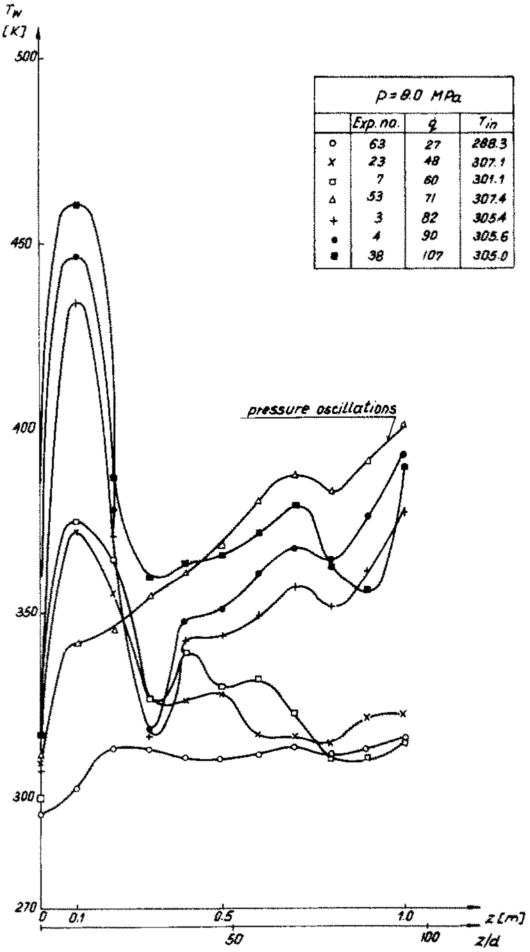

FIG. 4. Measured longitudinal wall temperature distribution at p = 8.0 MPa.

stress on the wall Re^* requiring simple numerical solution of an algebraic equation. In comparison to the other methods quoted in the References this method is very economical with respect to computer time—complete computations of one channel cross-section take only about 1 min on Polish Odra 1204 computer.

The property averaging procedure requires the wall temperature to be known *a priori* which is a shortcoming of the method when applied in designing. Actually, the wall temperature in technological devices is not known at the beginning. Thus, relevant iterative procedure, starting for example from one of the experimental formulae described in the References, would be of use in that case.

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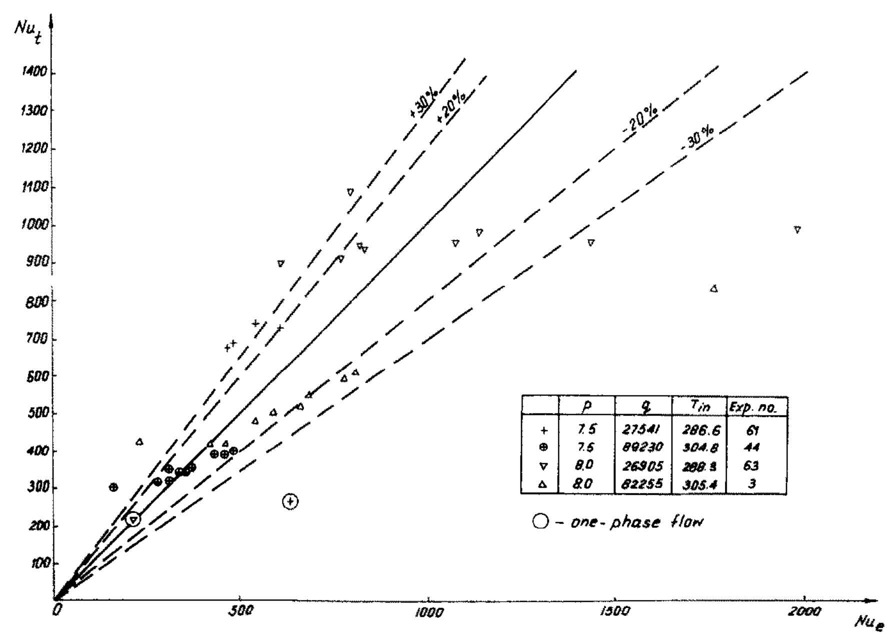

FIG. 5. Theoretical vs. experimental Nusselt numbers.

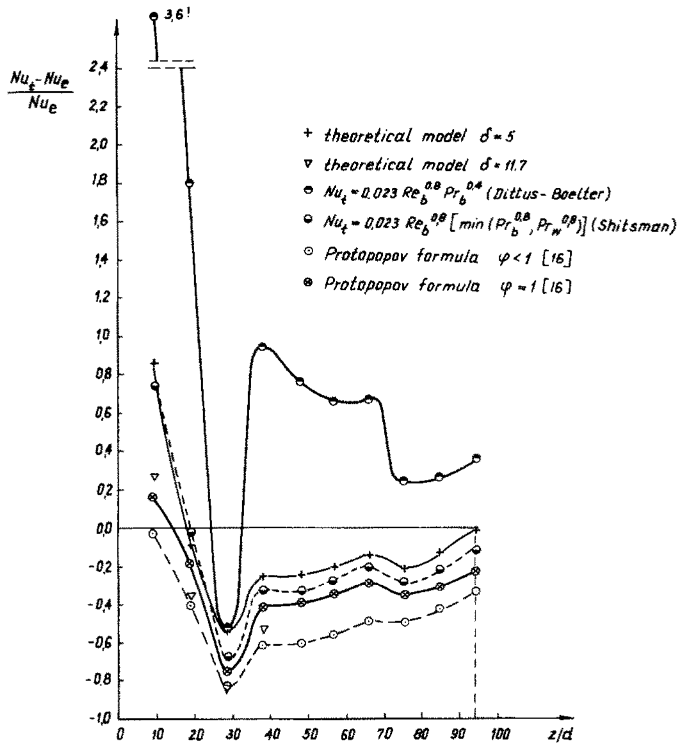

FIG. 6. Distribution of the relative Nusselt number error along the tube length for the model and three experimental formulae.

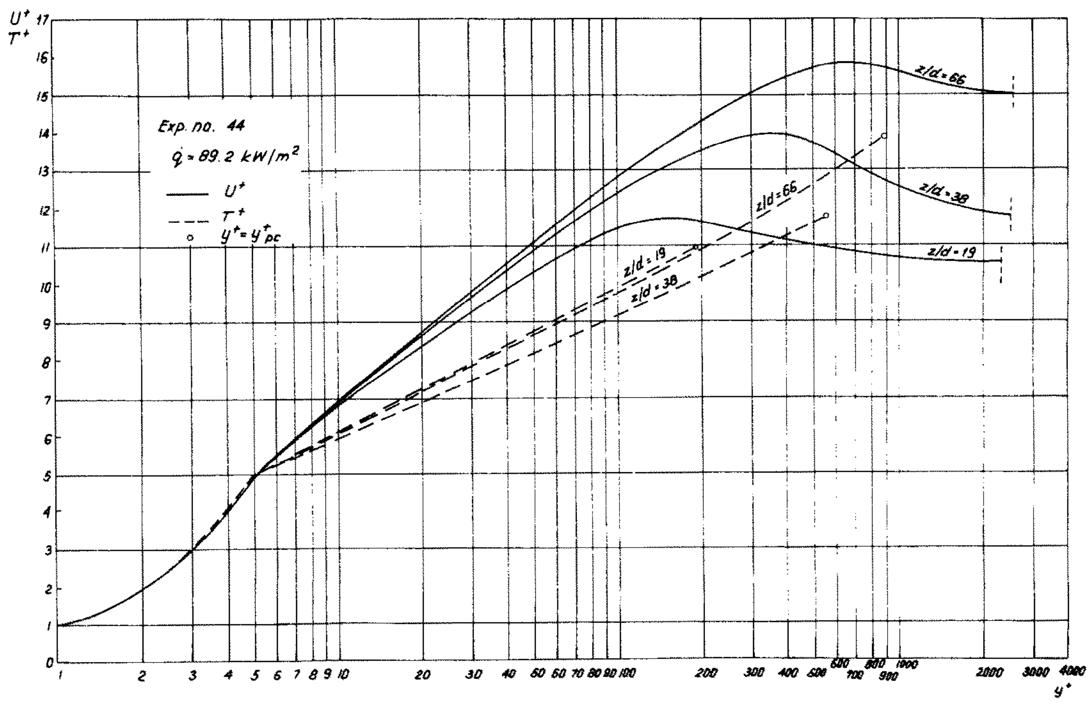

FIG. 7. Predicted dimensionless velocity and temperature profiles at high heat flux in selected cross-sections of the tube.

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TRANSFERT THERMIQUE DANS UN CANAL A PRESSION SUPERCRITIQUE

Résumé—On étudie théoriquement la convection thermique à des pressions supercritiques dans un canal. L'analyse est basée sur la division de l'écoulement en deux zones avec des propriétés moyennes, l'interface entre elles étant la surface de la température pseudo-critique. Les équations de bilan de quantité de mouvement et d'énergie peuvent être résolues séparément et conduisent à des formules analytiques pour les profils de vitesse, de température et pour le nombre de Nusselt. Les résultats théoriques sont comparés à des expériences avec CO_2 et ils montrent un accord satisfaisant.

WÄRMEÜBERTRAGUNG IN EINEM KANAL BEI ÜBERKRITISCHEM DRUCK

Zusammenfassung—Der Bericht befaßt sich mit einer theoretischen Analyse des konvektiven Wärmeübergangs bei überkritischen Drücken in einem Kanal. Die analytische Behandlung beruht auf der Unterteilung der Strömung in zwei Zonen mit gemittelten Eigenschaften, deren Grenzfläche bei der pseudokritischen Temperatur liegt. Daher dürfen die Bilanzgleichungen für Impuls und Energie separat gelöst werden, woraus analytische Gleichungen für Geschwindigkeits- und Temperaturprofile sowie die Nusselt-Zahl erhalten werden. Die theoretischen Ergebnisse werden neben eigenen experimentellen Ergebnissen, die mit CO_2 gewonnen wurden, dargestellt, wobei eine ziemlich gute Übereinstimmung festgestellt werden kann.

ТЕПЛОПЕРЕНОС В КАНАЛЕ ПРИ СВЕРХКРИТИЧЕСКОМ ДАВЛЕНИИ

Аннотация — Дан теоретический анализ конвективного теплопереноса при сверхкритическом давлении в канале. В основу анализа положено разделение потока на две области с осредненными характеристиками, граница раздела между которыми является поверхностью псевдокритической температуры, вследствие чего уравнения сохранения импульса и энергии могут решаться раздельно. В результате получены аналитические формулы для определения профилей скорости и температуры, а также числа Нуссельта. Сравнение теоретических и экспериментальных данных для CO_2 дает довольно хорошее совпадение результатов.